# The Structures of the $\boldsymbol{\gamma}$-Phases in the $\mathrm{Pd}-\mathrm{Cd}$ and $\mathrm{Pt}-\mathrm{Cd}$ Systems 

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#### Abstract

The primitive cubic $\gamma-\mathrm{Pd}_{8} \mathrm{Cd}_{43}$ and $\gamma-\mathrm{Pt}_{8} \mathrm{Cd}_{40}$ phases have structures similar to that of $\gamma$-brass-like $\mathrm{Ni}_{8} \mathrm{Cd}_{40}$, in that they have vacancies in the inner-tetrahedral positions. In $\mathrm{Pt}_{8} \mathrm{Cd}_{40}$, all Pt atoms have entered the outer-tetrahedral positions. Alloys of the approximate composition $T_{8} \mathrm{Cd}_{41}$ in both systems have bodycentered cubic superstructures with lattice parameters of $\sim 20 \AA$. The superstructure of $\mathrm{Pt}_{8} \mathrm{Cd}_{41}$ can be described in terms of three different 'clusters', all similar to $\gamma$-brass clusters, although two of them have lower symmetry. Atoms are totally absent from the inner-tetrahedral position of one type of cluster, while Cd atoms occupy that position in the others. The outer-tetrahedral positions of all clusters are occupied by Pt.


In the $\mathrm{Pd}-\mathrm{Cd}$ and $\mathrm{Pt}-\mathrm{Cd}$ systems there exist cubic phases with $\sim 83 \mathrm{at} . \% \mathrm{Cd}$; in both systems they are designated $\gamma$. Novotny, Stempfl \& Bittner (1951) (Pd-Cd) and Novotny, Bauer, Stempfl \& Bittner (1952) ( $\mathrm{Pt}-\mathrm{Cd}$ ) have suggested that the phases have $\gamma$-brass-like structures although the composition $T_{5} \mathrm{Cd}_{21}$, which could be expected from the HumeRothery rule, is outside the homogeneity range of both phases; these ranges are $80.7-83.3$ at. $\% \mathrm{Cd}$ for $\mathrm{Pd}-\mathrm{Cd}$, and $82 \cdot 5-85 \cdot 5 \mathrm{at} . \% \mathrm{Cd}$ for $\mathrm{Pt}-\mathrm{Cd}$. The authors have reported cubic lattice parameters, $a=$ 9.960 and $a=9.897 \AA$ for the $82.5 \% \mathrm{Cd}-\mathrm{Pd}$ and $83.9 \% \mathrm{Cd}-\mathrm{Pt}$ alloys, respectively. In the $\mathrm{Ni}-\mathrm{Cd}$ system there is a $\gamma$-brass-like phase in the same composition range. The structure of this phase, $\mathrm{Ni}_{8} \mathrm{Cd}_{40}$, has been determined by Ljung \& Westman (1970). The cubic unit cell, $a=9.7878 \AA$, is primitive and contains 48 atoms, leaving the inner-tetrahedral position of one cluster vacant. The present investigation was undertaken in order to establish relationships between the $\gamma$-phases of Cd and $\mathrm{Ni}, \mathrm{Pd}$ and Pt.

## Experimental

The starting materials, cadmium (very pure sticks, the British Drug Houses, $99.9 \% \mathrm{Cd}$ ), palladium (wire,

Koch-Light, $99.99 \%$ Pd) and platinum (dust, Johnson \& Mathews, spectroscopically pure), were weighed out to match several different compositions around $83 \mathrm{at} . \%$ Cd . The $\mathrm{Pd}-\mathrm{Cd}$ mixtures were heated together at 973 K and the Pt-Cd mixtures at 1013 K in evacuated silica capsules for 2 d . The alloys were then quenched in water, weighed and crushed. The alloys were reheated in the same manner: $\mathrm{Pd}-\mathrm{Cd}$ at 733 K for 12 weeks, $\mathrm{Pt}-\mathrm{Cd}$ at 933 K for 6 weeks, and then quenched in water. Subsequent weighing showed that the $\mathrm{Pt}-\mathrm{Cd}$ alloys had suffered some weight loss ( $\sim 0.5 \%$ ) presumably due to evaporation of Cd , which has the higher vapor pressure. The observation that the metallic deposits found in the top of the capsules were soft and not brittle like the rest of the preparations supported the assumption that they consisted of only Cd. The final compositions were calculated from the weighed-in amounts of the reactants, corrected for the loss of cadmium. No chemical analyses were performed, since it has been observed in other $\gamma$-brasses, synthesized as above, that the mole fractions calculated from the weighed-in amounts differ from those derived from chemical analysis only by a few units in the third decimal place (Westman, 1972). Density measurements were performed by weighing the alloys in air and in $\mathrm{CHCl}_{3}$.

Single crystals were picked from three of the samples. Subsequent comparisons between the diffraction patterns of the bulk powder materials and those of the corresponding single crystals showed that the compositions were the same. Crystal data are given in Table 1. The lattice parameters were calculated from Guinier photographs. Rotation and zero-level Weissenberg photographs were obtained from all three single crystals. Single-crystal X-ray data for $\mathrm{Pd}_{0.156} \mathrm{Cd}_{0.844}$ were collected on a Siemens automatic single-crystal diffractometer (SAED) with Mo $K \alpha$ radiation. 699 intensities were recorded in the range $5^{\circ} \leq \theta \leq 40^{\circ}$ with the $\theta-2 \theta$ scan technique, and were reduced to 343 independent structure factors. 213 of these, with $\sigma(I) / I$ $<0.4$, were used in the initial refinements.

The X-ray data from the single crystal of $\mathrm{Pt}_{0.168} \mathrm{Cd}_{0.832}$ were collected with Mo $K \alpha$ radiation on a Pailred linear single-crystal diffractometer controlled by an IBM 1800 computer. Intensities from 568 reflections with $5^{\circ} \leq \theta \leq 30.3^{\circ}$ were evaluated from

Table 1. Crystal data
Compound
Composition
Lattice parameter $a(\AA)$
Measured density $\left(\mathrm{Mg} \mathrm{m}^{-3}\right)$
Calculated density $\left(\mathrm{Mg} \mathrm{m}^{-3}\right)$
$Z$
Space group
Crystal size $(\mu \mathrm{m})$
Linear absorption coefficient $\mu\left(\mathrm{mm}^{-1}\right)$

| $\mathrm{Pd}_{8} \mathrm{Cd}_{43}$ | $\mathrm{Pt}_{8} \mathrm{Cd}_{40}$ | $\mathrm{Pt}_{8} \mathrm{Cd}_{41}$ |
| :---: | :---: | :---: |
| $\mathrm{Pd}_{0.156} \mathrm{Cd}_{0.844}=\mathrm{Pd}_{8} \mathrm{Cd}_{43.2}$ | $\mathrm{Pt}_{0.168} \mathrm{Cd}_{0.832}=\mathrm{Pt}_{8} \mathrm{Cd}_{39.6}$ | $\mathrm{Pt}_{0.164} \mathrm{Cd}_{0.836}=\mathrm{Pt}_{8} \mathrm{Cd}_{40.8}$ |
| $9.9415 \pm 5$ | $9.9200 \pm 3$ | $19.8042 \pm 9$ |
| $9.62 \pm 4$ | $10.38 \pm 4$ | $10 \cdot 51 \pm 7$ |
| 9.64 | $10 \cdot 30$ | 10.51 |
| 1 | 1 | 8 |
| P4̄3m | P ${ }^{4} 3 \mathrm{~m}$ | İ 3 3m |
| $50 \times 50 \times 30$ | $70 \times 30 \times 20$ | $100 \times 20 \times 20$ |
| 26.2 | $50 \cdot 6$ | 232.7 |

the Gaussian profile analysis described by Norrestam (1972). These intensities were then reduced to 149 independent structure factors.
For the compound $\mathrm{Pt}_{0.164} \mathrm{Cd}_{0.836}$, single-crystal film data were collected with an integrating Weissenberg camera using $\mathrm{Cu} K a$ radiation, and evaluated by means of an automatic SAAB film scanner-IBM 1800 computer system. A description of the program system has been published by Werner (1969). 1207 intensities were recorded and reduced to 333 independent structure factors.
A satisfactory absorption correction could only be applied on the $\mathrm{Pd}-\mathrm{Cd}$ alloy data. Attempts to correct the $\mathrm{Pt}-\mathrm{Cd}$ intensities for absorption did not decrease the differences between equivalent reflections.
In the structure determination the programs MULTAN (Germain, Main \& Woolfson, 1971), CRYM (Duchamp, 1964) and UPALS were used for phase determination, Fourier calculations and leastsquares calculations, respectively. Scattering factors were taken from Cromer \& Mann (1968) and dispersion corrections from Cromer \& Liberman (1970). The calculations were carried out on an IBM 370/165 computer.

## The $\boldsymbol{\gamma}$-Pd-Cd structure

Two of the $\mathrm{Pd}-\mathrm{Cd}$ preparations were single phase. The alloy with the composition $\mathrm{Pd}_{0.156} \mathrm{Cd}_{0.844}$ showed a primitive cubic Guinier pattern with a lattice parameter $a=9.9415 \pm 5 \AA$. At a somewhat lower Cd concentration, $\overline{\mathrm{Pd}}_{0.167} \mathrm{Cd}_{0.833}$, the lattice parameter was $a=9.9417 \pm 6 \AA \AA$. This powder pattern also showed a few very weak lines which could be indexed as superstructure reflections of a body-centered cubic cell with a doubled lattice parameter: $a=19.8834 \pm 8 \AA$. A single crystal was picked of $\mathrm{Pd}_{0.156} \mathrm{Cd}_{0.844}$ only, since the additional reflections from the other sample were too weak to be measured with the accuracy needed to solve the superstructure. The rotation and Weissenberg photographs showed no superstructure reflections, and they indicated the Laue symmetry $m 3 m$.

In order to obtain an independent starting model for the structure refinement, the data set was phased by
direct methods in space group $P \overline{4} 3 m$. Only the strongest reflections, with $h+k+l=2 n$, could be phased accurately; consequently the structure model arrived at was body-centered. This model was indeed a normal $\gamma$-brass-like structure, but contained a split octahedral position: $z_{\mathrm{OH}}=0.36$ and 0.33 . The starting model for refinement of the primitive cell was one with two identical $\gamma$-brass-like clusters, except that cluster $A$, centered on $0,0,0$, had the smaller octahedron $\left[z_{\mathrm{OH}(A)}=\right.$ $0 \cdot 33$ ] and cluster $B$, around $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$, had the larger one $\left[z_{\mathrm{OH}(\mathrm{B})}=\frac{1}{2}+0 \cdot 36\right]$. Both clusters contained Cd at all sites. A refinement of this model converged to an $R$ value of $10.8 \%$. The value of the temperature factor $B_{\mathrm{IT}(B)}$ was somewhat higher than the others: $2.7 \AA^{2}$ compared to $0.2-1.5 \AA^{2}$.

Since the composition and density indicate the presence of one Cd vacancy per unit cell, the next model chosen for refinement was one with only three of the four $\operatorname{IT}(B)$ sites occupied. This refinement converged to $R=10.4 \% . B_{\mathrm{IT}(B)}$ decreased to $1 \cdot 1 \AA^{2}$ while the rest of the parameters remained generally unaffected. In a recent investigation by Arnberg, Hovmöller \& Westman (1979), it was recommended that the weak reflections should also be included in the final refinement of a structure from which data have been collected in a constant-time diffractometer experiment. In the next refinement the 130 weakest reflections with $\sigma(I) /(I) \geq 0.4$ were therefore included. The $R$ value increased to $14.4 \%$, the standard deviations of the positional parameters decreased, albeit very slightly, and some of the atoms shifted by more than one standard deviation. $B_{\operatorname{IT}(A)}$ increased to $2.9 \dot{\AA}^{2}$ while $B_{\mathrm{IT}(B)}$ decreased to $0.3 \AA^{2}$. The result clearly indicated that the structure model obtained from the strong reflections was wrong and that the vacancy should occur in cluster $A$ instead of $B$. This is, moreover, a more reasonable model, since there is less space in cluster $A$ due to the smaller octahedron. A refinement from all reflections, with the vacancy in cluster $A$, converged to $R=13 \cdot 2 \%$. The temperature factors of $\operatorname{IT}(A)$ and $\operatorname{IT}(B)$ were now 1.6 and $1.3 \AA^{2}$, respectively and the standard deviations of the positional coordinates were about $60 \%$ of those from the refinement with only the strong reflections.

Attempts were also made to refine structure models with either $\operatorname{IT}(A)$ or $\operatorname{IT}(B)$ completely empty by analogy with the $\mathrm{Ni}_{8} \mathrm{Cd}_{40}$ structure. These refinements gave $R$ values around $20 \%$, and the $B_{\mathrm{OH}}$ of the cluster without IT atoms became very high ( $\sim 15 \AA^{2}$ ). The data quality did not permit determination of the Pd sites, i.e. no attempts to substitute Cd for Pd at any of the sites improved the refinement results significantly. This is not surprising, considering the small difference in scattering power between Pd and Cd . A neutron diffraction experiment would certainly establish the Pd sites, but this requires more material than was available.

Table 2. Refined structural parameters of $\mathrm{Pd}_{8} \mathrm{Cd}_{43}$
Estimated standard deviations are given in parentheses in units of the last digit. Thermal parameters are in $\AA^{2}$.

|  |  | Cluster $A$ | Cluster $B$ |
| :---: | :---: | :---: | :---: |
| IT 4(e) $x$ x $x$ | $x$ | $\begin{gathered} \frac{3}{4} \mathrm{Cd} \\ 0.1048(9) \end{gathered}$ | $\begin{gathered} \mathrm{Cd} \\ 0.6073(6) \end{gathered}$ |
| Inner tetrahedron | B | 1.6 (2) | 1.3 (1) |
| OT 4(e) $x x x$ | $x$ | $\begin{gathered} \mathrm{Pd} \\ -0.1734(6) \end{gathered}$ | $\begin{gathered} \mathrm{Pd} \\ 0.3292(5) \end{gathered}$ |
| Outer tetrahedron | $B$ | 0.8 (1) | 0.4 (1) |
|  |  | Cd | Cd |
| OH 6(g) 00z | $z$ | $0 \cdot 3493$ (9) |  |
| 6(f) $\frac{1}{2} z$ | $z$ |  | 0.8556 (7) |
| Octahedron | B | 1.5 (1) | 0.7 (1) |
|  |  | Cd | Cd |
| CO 12(i) $x x z$ | $x$ | 0.3038 (4) | 0.8076 (4) |
| Cubo-octahedron | $z$ | 0.0474 (6) | 0.5448 (5) |
|  | B | 1.1 (1) | 1.0 (1) |

Table 3. Coordination numbers of $\mathrm{Pd}_{8} \mathrm{Cd}_{43}, \mathrm{Pt}_{8} \mathrm{Cd}_{40}$ and $\mathrm{Pt}_{8} \mathrm{Cd}_{41}$
$\mathrm{Cd}-\mathrm{Cd}$ distances $<3.5 \AA$ and $\mathrm{Cd}-\mathrm{Pd}, \mathrm{Pt}$ distances $<3.1 \AA$ have been counted. $\gamma$ - Brass and $\mathrm{Ni}_{8} \mathrm{Cd}_{40}$ are included for comparison.

|  | $\mathrm{Pd}_{8} \mathrm{Cd}_{43}{ }^{*}$ | $\mathrm{Pt}_{8} \mathrm{Cd}_{40}$ | $\mathrm{Pt}_{8} \mathrm{Cd}_{41}{ }^{*}$ |  |  |  | $\gamma$ - $\mathrm{Brass}^{c}$ |  | $\mathrm{Ni}_{8} \mathrm{Cd}_{40}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $A$ | $B$ | $A$ | $B$ | $A$ | $C$ | $B$ |  | $A$ | $B$ |
| IT1 | 12 | 12 |  | 12 | 12 |  | 12 | 12 |  | 12 |
| IT2 |  |  |  |  |  |  | 12 |  |  | 12 |
| OT1 | 12 | 12 | 9 | 12 | 12 | 9 | 12 | 12 | 9 | 12 |
| OT2 |  |  |  |  |  |  | 12 |  |  |  |
| OH1 | 13 | 13 | 11 | 13 | 12 | 8 | 13 | 13 | 12 | 13 |
| OH2 |  |  | 8 |  |  | 10 | 11 |  |  |  |
| CO1 | 11 | 11 | 12 | 10 | 11 | 12 | 10 | 11 | 12 | 10 |
| CO2 |  |  |  |  |  | 11 | 11 |  |  |  |
| CO3 |  |  |  |  |  |  | 11 |  |  |  |

[^0]The positional parameters given in Table 2* and the coordination numbers in Table 3 originate from the 51 atom/cell model described above.

A calculation of the mean distance from each atom to its nearest neighbors gives the three smallest values of $2.894,2.901$ and $2.908 \AA$ for the $\operatorname{IT}(A), \operatorname{OT}(A)$ and $\mathrm{OT}(B)$ atoms, respectively. It is reasonable to assume that the eight smaller Pd atoms enter two of these three sites. Westman (1972) pointed out that there is a tendency in $\gamma$-brasses for the transition-metal atoms to concentrate at the OT sites. This is also the case for $\gamma-\mathrm{Pt}-\mathrm{Cd}$; the small difference in atomic radius between Pd and $\mathrm{Pt}, r_{\mathrm{Pd}}=1.37 \AA$ and $r_{\mathrm{Pt}}=1.39 \AA$, implies that the distribution of the transition-metal atoms is similar. The assumption that the Pd atoms occupy the OT sites leaves some rather short $\mathrm{Cd}-\mathrm{Cd}$ distances in the structure. The shortest, three $\mathrm{IT}(A)-\mathrm{OH}(A)$ and three $\mathrm{IT}(A)-\mathrm{CO}(A)$ are 2.842 and $2.856 \AA$, respectively. These six short contacts occur, as could be expected, around $\operatorname{IT}(A)$ which is only partially occupied. There is another short contact, $\mathrm{OH}(B)-\mathrm{OH}(B)(2.871 \AA)$, which also occurs in the $\mathrm{Ni}_{8} \mathrm{Cd}_{40}$ structure. This is, however, a single contact in a coordination polyhedron where the mean distance from $\mathrm{OH}(B)$ to the 13 neighbors is $3.059 \AA$.

## The $\boldsymbol{\gamma}$ - $\mathrm{Pt}-\mathrm{Cd}$ structure

The Guinier powder diffraction patterns from the $\mathrm{Pt}-\mathrm{Cd}$ alloys were very similar to those from the $\mathrm{Pd}-\mathrm{Cd}$ system. In the $\mathrm{Pt}-\mathrm{Cd}$ system the two alloys were also single phase. One of them, with the composition $\mathrm{Pt}_{0.168} \mathrm{Cd}_{0.832}$, had a primitive cubic lattice with $a=9.9200 \pm 3 \AA$; and the other, with the composition $\mathrm{Pt}_{0.164} \mathrm{Cd}_{0.836}$, had a body-centered cubic superstructure with $a=19.8042 \pm 9 \AA$. The superstructure reflections were, however, stronger than those from the corresponding $\mathrm{Pd}-\mathrm{Cd}$ alloy, and hence single crystals were picked for both $\mathrm{Pt}-\mathrm{Cd}$ alloys. Weissenberg photographs of $\mathrm{Pt}_{0.164} \mathrm{Cd}_{0.836}$ showed observable superstructure reflections, while $\mathrm{Pt}_{0.168} \mathrm{Cd}_{0.832}$ films did not.

The structure determination of $\mathrm{Pt}_{0.168} \mathrm{Cd}_{0.832}$ followed the procedure of the $\gamma-\mathrm{Pd}-\mathrm{Cd}$ determination. The body-centered structure model obtained by direct methods was $\gamma$-brass-like, with a high OT peak indicating Pt at this site, and two peaks at OH : one at $z=0.35$ and the other at $z=0.26$.

The starting model for refinement of the primitive cell had one $\gamma$-brass cluster, $A$, around $0,0,0$, with the smaller octahedron $\left[z_{\mathrm{OH}(A)}=0.26\right]$ and an empty inner

[^1]tetrahedron as in $\mathrm{Ni}_{8} \mathrm{Cd}_{40}$, and another cluster, $B$, around $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$, containing the larger octahedron $\left[z_{\mathrm{OH}(B)}=\right.$ $0.35]$ and an inner tetrahedron occupied by Cd . The positional parameters of the OT and CO sites were identical in $A$ and $B$, and the OT sites of both clusters were occupied by Pt.

The vacancy concentration, $\sim 4 \mathrm{Cd}$ atoms absent/cell, was calculated from the composition and the density. A refinement of the starting model converged to $R=14.4 \%$. The thermal parameter of $\mathrm{OH}(A)$ was large ( $4.5 \AA^{2}$ ) compared to those of the other atoms, which ranged from -0.2 to $1.8 \AA^{2}$, indicating either too high an assumed occupancy or disorder at this site. A difference Fourier map revealed two peaks at the $\mathrm{OH}(A)$ site, one of them at $z=0.35$ and the other at $z=0.25$. This indication of disorder at the $\mathrm{OH}(A)$ site led to a model in which the $\mathrm{OH}(A)$ atoms are situated either at $z_{\mathrm{OH}(A)}=0.25$ or $z_{\mathrm{OH} 2(A)}=0.35$. A model with one half of the $\mathrm{OH}(A)$ atoms at $\mathrm{OH1}(A)$ and the remainder at $\mathrm{OH} 2(A)$ converged to an $R$ value of $12.0 \%$. After removal of the three strongest reflections, which were found to suffer from extinction, the occupancy parameters of $\mathrm{OH1}(A)$ and $\mathrm{OH} 2(A)$ were refined. This resulted in an occupancy of $3.6 \pm 0.8$ atoms at $\mathrm{OH} 1(A)$ and $2.4 \pm 0.6$ atoms at $\mathrm{OH} 2(A)$. The temperature factors were now not unduly scattered $(0.4-2.7)$, and the $R$ value was $10.7 \%$. All attempts to refine structure models with atoms at the $\operatorname{IT}(A)$ site or at the origin resulted in temperature factors in the region of $10 \AA^{2}$ for these atoms. The positional parameters of the disordered structure model with the composition $\mathrm{Pt}_{8} \mathrm{Cd}_{40}$ are given in Table 4* and the corresponding coordination numbers in Table 3.

> * See deposition footnote.

Table 4. Refined structural parameters of $\mathrm{Pt}_{8} \mathrm{Cd}_{40}$
Estimated standard deviations are given in parentheses in units of the last digit. Thermal parameters are in $\AA^{2}$.

| IT 4(e) $x x x$ Inner tetrahedron | $B$ | Cluster $A$ <br> Vacant | Cluster $B$ |
| :---: | :---: | :---: | :---: |
|  |  |  | 0.610 (2) |
|  |  |  | $2 \cdot 7$ (6) |
| OT 4(e) $x x x$ | $x$ | $\begin{gathered} \mathrm{Pt} \\ -0.1832(8) \end{gathered}$ | $\begin{gathered} \mathrm{Pt} \\ 0.3312(9) \end{gathered}$ |
| Outer tetrahedron | B | 1.3 (3) | 0.4 (2) |
|  |  | 0.6 Cd | Cd |
| OH1 6(g) 00z | $z$ | 0.337 (6) |  |
| $6(f) \frac{12}{22}$ | $z$ |  | 0.856 (2) |
| Octahedron | B | 1.6 (8) | 1.0 (3) |
|  |  | 0.4 Cd |  |
| OH2 6(g) 00z | $z$ | $0 \cdot 260$ (4) |  |
| Octahedron | B | $2 \cdot 3$ (7) |  |
|  |  | Cd | Cd |
| CO 12(i) $x$ xz | $x$ | 0.296 (1) | 0.818 (1) |
| Cubo-octahedron | $z$ | 0.051 (1) | 0.536 (2) |
|  | B | 1.0 (2) | $2 \cdot 3$ (4) |

The starting model for refinement of the $\mathrm{Pt}_{0.164} \mathrm{Cd}_{0.836}$ superstructure in space group $I \overline{4} 3 m$ was derived from the $\mathrm{Pt}_{8} \mathrm{Cd}_{40}$ structure as follows: the cell contains sixteen clusters of which two identical ones, $A$, are centered on $0,0,0$ etc.; six other identical clusters, $C$, are grouped around $0, \frac{1}{2}, \frac{1}{2}$ etc., and eight identical clusters, $B$, are centered on $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$ etc. The $B$ and $C$ clusters will, of course, have lower symmetry than the ordinary $\gamma$-brass-like cluster. Clusters $A$ and $C$ of the superstructure will then correspond to cluster $A$ of the primitive subcell, and cluster $B$ of the superstructure to cluster $B$ of the subcell. The model was based on the assumption that clusters $A$ and $C$ differ with respect to their octahedra, as was indicated by the disorder of the subcell. It was further assumed that the larger subcell octahedron, with the lowest occupancy factor, should occur in cluster $A$.

Table 5 shows the relationship between the starting structure model of $\mathrm{Pt}_{0.164} \mathrm{Cd}_{0.836}$ and the structure of $\mathrm{Pt}_{8} \mathrm{Cd}_{40}$. A refinement of this model converged to $R=$ $17.6 \%$. A difference Fourier map at this stage showed a peak at $x, x, x, \simeq 0.05$, indicating that the $\operatorname{IT}(A)$ site

Table 5. Relationships between positional parameters in the superstructure of $\gamma-\mathrm{Pt}-\mathrm{Cd}$ and the structure of $\mathrm{Pt}_{8} \mathrm{Cd}_{40}$

Site in the superstructure

| Cluster $A$ centered on 2(a) at 0,0,0 | $\begin{array}{lr} \text { OT } & 8(c) x, x, x \\ \text { OH } & 12(d) 0,0, z \\ \text { CO } & 24(g) x, x, z \end{array}$ | $\begin{aligned} & x=\frac{1}{2} x_{\mathrm{OT}(A)}{ }^{*} \\ & z=\frac{1}{2} x_{\mathrm{OH} 2(A)} \\ & x=\frac{1}{2} x_{\mathrm{CO}(A)} \\ & z=\frac{1}{2} z_{\mathrm{CO}(A)} \end{aligned}$ |
| :---: | :---: | :---: |
| Cluster $C$ centered on 6(b) at at $0, \frac{1}{2}, \frac{1}{2}$ | $\begin{aligned} & \text { OT } 24(g) x, x, z \\ & \text { OH1 } 12(d) 0,0, z \\ & \text { OH2 } 24(f) x, 0, \frac{1}{2} \\ & \text { CO1 } 24(g) x, x, z \\ & \text { CO2 } 48(h) x, y, z \end{aligned}$ | $\begin{aligned} & x=\frac{1}{2} x_{\mathrm{OT}(A)} \\ & z=\frac{1}{2}\left[1-x_{\mathrm{OT}(A)}\right] \\ & z=\frac{1}{2}\left[1+x_{\mathrm{OH}(A)}\right] \\ & x=\frac{1}{2} x_{\mathrm{OH}(A)} \\ & x=\frac{1}{2} x_{\mathrm{CO}(A)} \\ & z=\frac{1}{2}\left[1+z_{\mathrm{CO}(A)}\right] \\ & x=\frac{1}{2} x_{\mathrm{CO}(A)} \\ & y=\frac{1}{2} z_{\mathrm{COO}(A)} \\ & z=\frac{1}{2}\left[1+x_{\mathrm{CO}(A)}\right] \end{aligned}$ |
| Cluster $B$ centered on $8(c)$ at $\frac{1}{4}, 4,4$ |  |  |

Table 6. Refined structural parameters of $\mathrm{Pt}_{8} \mathrm{Cd}_{41}$
Estimated standard deviations are given in parentheses in units of the last digit. Thermal parameters are in $\AA^{2}$.

| Cluster $A$ |  |  | Cluster $B$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1 T$ | 0.8 Cd |  | ITI | 8(c) $x x x$ | Cd |
|  | $8(c) x x x$ | $\begin{array}{ll} x & 0.043(5) \\ B & 4(3) \end{array}$ |  |  | $\begin{array}{cc} x & 0.299(1) \\ B & -2.3(6) \end{array}$ |
| OT | 8(c) $x \times x$ | $\mathrm{Pt}^{\text {t }}$ | IT2 | $24(g) x x z$ | Cd |
|  |  | $x-0.0840$ (5) |  |  | $0.194(1)$ $0.314(2)$ |
|  |  | $B \quad-3.2(2)$ |  |  | $\begin{array}{cc}2 & 0.314(2) \\ B & 0.10(5)\end{array}$ |
| OH |  | 0.178(2) | OTI | 8(c) $x x x$ | $\mathrm{Pt}_{\mathrm{t}}$ |
|  | 12(d) 00z | $\begin{array}{ll}\text { B } & -1.0(6)\end{array}$ |  |  | - $\begin{gathered}0.1699 \\ -1.2(9)\end{gathered}$ |
|  |  | Cd |  | $24(\mathrm{~g}) \times x z$ |  |
| co | $24(g) x x z$ | $x \quad 0.151(1)$ | OT2 |  | 0.3343 (7) |
|  |  | ${ }^{2} \mathrm{~B}_{\mathrm{B}} \mathrm{O}_{0}^{0.023(1)}$ |  |  | $\begin{aligned} & 0.3343(7) \\ & 0.164(1) \end{aligned}$ |
|  |  | B -1.8 (3) |  |  | $\begin{array}{ll}\text { B } & -1.6 \text { (2) }\end{array}$ |
|  | Cluster $C$ |  | Cd |  |  |
| OT | $24(g) x x z$ | Pt | OH | $24(g) x x z$ | 0.247 (1) |
|  |  | $x$-0.0985 (6) |  |  | 0.428 (2) |
|  |  | 0.4112 (8) |  |  | $B-2.2(3)$ |
|  |  | -1.2(3) |  | $24(g) x x z$ | Cd |
| $\mathrm{OH1}$ | 12(d) 00z | Cd | OH2 |  | 0.255 (1) |
|  |  | 0.627 (2) |  |  | 0.069 (3) |
|  |  | B -0.2(7) |  |  | 0.4 (6) |
| OH2 | $24(f) \times 0 \frac{1}{2}$ | Cd | COI | Cd |  |
|  |  | 0.158 (3) |  | 24(g) $x x z$ | 0.411 (1) |
|  |  | $B \quad 2.0$ (7) |  |  | $0.266(1)$ |
|  |  | Cd |  |  | B -1.6(5) |
| COL | 24(g) $x x z$ | $x \quad 0.149$ (1) | CO2 | $24(g) x x z$ | Cd |
|  |  | $0.521(1)$ |  |  | $x \quad 0.096$ (1) |
|  |  | $B-2.0$ (3) |  |  | $z \quad 0.270$ (2) |
|  |  | Cd |  |  | $B \quad-0.2(6)$ |
| CO2 | 48(h) $x y z$ | $x \quad 0 \cdot 140$ (1) |  |  | Cd |
|  |  | $y \quad 0.032(1)$ | CO3 | 48(h) $x y z$ | $x \quad 0.229$ (2) |
|  |  | $z \quad 0.642(\mathrm{I})$ |  |  | $y \quad 0.097(\mathrm{I})$ |
|  |  | $B \quad-0.4(4)$ |  |  | $\begin{array}{ll}z & \\ B & 0.414(1) \\ -0.5(4)\end{array}$ |

should be occupied. Since the observed composition and density required a somewhat higher cell content of cadmium, the next model to be refined had the IT $(A)$ site occupied by Cd. A few of the other atoms in the model were given slight initial displacements according to the difference Fourier map, and before the refinement the three strongest reflections were deleted since they were appreciably affected by extinction.

The model refined to an $R$ value of $14.6 \%$. The temperature factor of $\mathrm{IT}(A)$ was large ( $5.6 \AA^{2}$ ) compared to the others which ranged from -3.3 to 1.3 $\AA^{2}$. The occupancy factor of $\operatorname{IT}(A)$ was therefore refined; it converged to $2.9 \pm 0.8$ atoms $/ \mathrm{IT}(A)$ site. The value of $B_{\mathrm{IT}(A)}$ decreased to $4.3 \AA^{2}$ and the $R$ value to $14.3 \%$. Several attempts were made to refine models with atoms differently distributed around the origin, at the origin and at the $\operatorname{IT}(C)$ site, but they yielded temperature factors of $10 \AA^{2}$ or more for the corresponding atoms. The model with the partially occupied $\operatorname{IT}(A)$ site was thus considered to be the best one, although the large temperature factor of $\operatorname{IT}(A)$, the corresponding positional parameter of which gives $\mathrm{Cd}-\mathrm{Cd}$ distances of $2.4 \AA$, and the large standard deviation of $x_{\mathrm{IT}(A)}$ clearly indicate disorder around the origin.

The positional parameters are given in Table 6* and the coordination numbers in Table 3. There are some rather short $\mathrm{Cd}-\mathrm{Cd}$ distances in the structure, namely $\mathrm{OH}(A)-\mathrm{COl}(B)$ and $\operatorname{IT} 2(B)-\mathrm{OH1}(B)$ of 2.72 and $2.69 \AA$, respectively. These are, however, single bonds in coordination polyhedra with considerably longer mean $\mathrm{Cd}-\mathrm{Cd}$ distances. The shortest $\mathrm{Pt}-\mathrm{Cd}$ distances are between $\mathrm{OT}(B)$ and $\mathrm{CO} 3(B)(2.58 \AA)$, and $\mathrm{OT}(B)$ and $\operatorname{CO1}(C)(2.59 \AA)$.
As can be seen from Table 3, the temperature factors are rather scattered [even disregarding $B_{\mathrm{IT}(A)}$ ] and many of them have negative values. This is probably a consequence of the lack of absorption correction. A better determination of the structure could certainly be obtained from a single crystal prepared according to the procedure described by Arnberg \& Westman (1978).

## Discussion of the structures

There are several similarities between the structures of $\gamma-\mathrm{Ni}-\mathrm{Cd}, \gamma-\mathrm{Pd}-\mathrm{Cd}$ and $\gamma-\mathrm{Pt}-\mathrm{Cd}$, although the transi-tion-metal distribution is not the same in $\gamma-\mathrm{Ni}-\mathrm{Cd}$ as in $\gamma-\mathrm{Pd}-\mathrm{Cd}$ and $\gamma-\mathrm{Pt}-\mathrm{Cd}$. One common feature is the presence of vacancies in the inner tetrahedron of one of the clusters. The number of atoms at this site seems to vary from zero to at least three.

The distortion of the structures in certain vacancyconcentration ranges causes cubic superstructures with doubled cell parameters. This feature has also been observed in the $\mathrm{Ni}-\mathrm{Cd}$ system by Lihl \& Buhl (1955), who found a $\gamma$-brass-like alloy with $a=19.545 \AA$ at $18.5 \mathrm{at} . \% \mathrm{Ni}$. The superstructure model suggested for $\gamma \mathrm{Pt}-\mathrm{Cd}$ permits ideally ordered structures at the compositions $T_{8} \mathrm{Cd}_{41}$ [with $\operatorname{IT}(A)$ occupied and $\operatorname{IT}(C)$ vacant] and $T_{8} \mathrm{Cd}_{43} \quad[\mathrm{IT}(A)$ vacant and $\operatorname{IT}(B)$ occupied]. It is possible that $\mathrm{Pd}_{8} \mathrm{Cd}_{43}$ represents the latter structure type, although the superstructure reflections were too weak to be observed.

As can be seen from Table 3, the structure most similar to $\gamma$-brass is $\mathrm{Pd}_{8} \mathrm{Cd}_{43}$. This is quite natural, since the number of vacancies is low (one per unit cell).

The structures of $\mathrm{Pt}_{8} \mathrm{Cd}_{40}$ and $\mathrm{Ni}_{8} \mathrm{Cd}_{40}$ differ with respect to the $\mathrm{OH}(A)$ atom. Four $\mathrm{OH}(A)-\mathrm{OH}(A)$ contacts occur in $\mathrm{Ni}_{8} \mathrm{Cd}_{40}$. These are mainly $\mathrm{Ni}-\mathrm{Ni}$ distances of $3.471 \AA$. In $\mathrm{Pt}_{8} \mathrm{Cd}_{40} \mathrm{OH}(A)$ is occupied by Cd , and the corresponding distance $[\mathrm{OH} 2(A)-$ $\mathrm{OH} 2(A)$ ] is consequently longer ( $3 \cdot 653 \AA$ ).

The $A$ cluster of the $\mathrm{Pt}_{8} \mathrm{Cd}_{41}$ structure can be regarded as a normal $\gamma$-brass cluster without the $\mathrm{OH}(A)-\mathrm{OH}(A)^{\prime}$ contact. This distance [here $\mathrm{OH}(A)-$ $\mathrm{OH1}(C)]$ is $3.831 \AA$. The $C$ cluster of this structure is very similar to the corresponding cluster of the subcell $\left(\mathrm{Pt}_{8} \mathrm{Cd}_{40}\right.$ cluster $\left.A\right)$. $\mathrm{OH} 2(C)$ has, however, two contacts to $\mathrm{CO} 2(B)$ which do not occur for the

[^2]$\mathrm{OH} 2(A)$ atom of $\mathrm{Pt}_{8} \mathrm{Cd}_{40}$. The $B$ clusters of the two $\mathrm{Pt}-\mathrm{Cd}$ structures are also very similar. The two $\mathrm{OH}(B)-\mathrm{CO}(B)$ contacts are lost for one of the atoms [OH2(B)], reducing the CN for this atom from 13 to 11. Two of the $\mathrm{CO}(B)$ atoms in $\mathrm{Pt}_{8} \mathrm{Cd}_{41}$ have $\mathrm{CO}(B)-\mathrm{OH}(B)$ contacts which are longer ( $2 \cdot 739 \AA$ ) in the $\mathrm{Pt}_{8} \mathrm{Cd}_{40}$ structure.

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# Structure Reinvestigation of the High-Temperature Form of $\mathbf{K}_{\mathbf{2}} \mathbf{S O}_{\mathbf{4}}$ 

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#### Abstract

The crystal structure of the high-temperature form of $\mathrm{K}_{2} \mathrm{SO}_{4}$ was redetermined from three-dimensional X-ray diffractometer data at 1073 K and refined to an $R$ value of 0.086 for 66 observed reflexions. The structure belongs to the hexagonal space group $\mathrm{Pb}_{3} / \mathrm{mmc}$, with $a=5.947$ (2), $c=8.375$ (3) $\AA, Z=2$, and $D_{x}=2.26$ $\mathrm{Mg} \mathrm{m}{ }^{-3}$. The $\mathrm{SO}_{4}^{2-}$ tetrahedron has two orientations with one of the apices of the $\mathrm{SO}_{4}^{2-}$ tetrahedron pointing statistically in opposite directions along the $c$ axis. The net entropy change, $\Delta S=5.02 \mathrm{~J} \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$ at the phase-transition point ( 860 K ), was explained successfully by the configurational change of the $\mathrm{SO}_{4}^{2-}$ tetrahedron in the low- and high-temperature forms. The $\mathrm{SO}_{4}^{2-}$ tetrahedron is approximately undistorted with a corrected mean $\mathrm{S}-\mathrm{O}$ distance of $1.43 \AA$. Two crystallographically independent K atoms are coordinated by 9 and 13 O atoms with mean $\mathrm{K}-\mathrm{O}$ distances of 3.20 and $3.37 \AA$, respectively.


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## Introduction

Two modifications of potassium sulphate, $\mathrm{K}_{2} \mathrm{SO}_{4}$, were reported by Bredig (1942), Bernard \& Hocart (1961) and Fischmeister (1962). The first-order phase transition between them occurs at 860 K . The space group and structure of the low-temperature form are well established (Robinson, 1958; McGinnety, 1972).

The space groups reported for the high-temperature form of $\mathrm{K}_{2} \mathrm{SO}_{4}$ were inconsistent. Bredig (1942) and Eysel \& Hahn (1970) proposed the space groups $P \overline{3} m 1$ and $P 6_{3} m c$, respectively, from powder X-ray diffraction analyses. Iwai, Sakai \& Watanabé (1973) carried out a structure analysis using a high-temperature Weissenberg technique. They concluded that the space group is $P 6_{3} m c$, but ambiguity in the structure remains unexplored. The ferroelastic study of Shiozaki, Sawada, Ishibashi \& Takagi (1977) suggests that the space group is $P 6_{3} / \mathrm{mmc}$.

Recently, van den Berg \& Tuinstra (1978) refined the © 1980 International Union of Crystallography


[^0]:    * Coordination numbers have been calculated as though all positions were completely occupied. For $\mathrm{Pd}_{8} \mathrm{Cd}_{43}$ and $\mathrm{Pt}_{8} \mathrm{Cd}_{41}$, with partially vacant $\mathrm{IT}(A)$ sites, some of the actual CN values are slightly lower.

[^1]:    * Lists of structure factors for the three structures have been deposited with the British Library Lending Division as Supplementary Publication No. SUP 34935 ( 8 pp .). Copies may be obtained through The Executive Secretary, International Union of Crystallography, 5 Abbey Square, Chester CH1 2HU, England.

[^2]:    * See deposition footnote.

